Motion estimation from satellite image sequences: validation

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1 Introduction

The issue of surface velocity estimation from satellite images has been extensively studied in the literature [1, 2, 3, 4, 5, 6, 7]. Data Assimilation (DA) techniques have been applied in the last five years and gain importance in the scientific community [8, 9, 10, 11]. The key points of the DA approach are: availability of heuristics on the dynamics of a satellite sequence, knowledge on links between velocity and image data.

This paper proposes an analysis and a validation of the DA approach for motion estimation from ocean satellite images. Two *Image Models* were proposed in [12, 13, 14]. They express heuristics on the dynamic of the motion field. The comparison of the estimation using the two models allows us to analyze the impact of these heuristics. The main issue of the paper is then to validate the estimation approach by evaluating the quality of the result, compared to real data.

The motion estimation is performed with NOAA/AVHRR Sea Surface Temperature (SST) data acquired over the Black Sea. The analysis is conducted by comparing the stationary and the shallow-water heuristics. The validation is obtained by quantifying the discrepancy of the water layer thickness, estimated with the shallow-water image model, and the one computed from altimetry data. The altimetry measures, used in this study, come from the Envisat and GFO sensors.

The paper is organized as follows. Section 2 summarizes the principles of variational data assimilation. The definition of the *Stationary Image Model* (SIM) and *Shallow Water Image Model* (SWIM) is given in Section 3. Section 4 describes the application of DA to perform motion estimation. Section 5 describes the SST images (5.1), displays and analyzes the estimated motion result (5.2), describes the altimetry data (5.3), and validates the approach (5.4).

2 Variational Data Assimilation

2.1 Mathematical setting

Let $\mathbf{X}$ being the state vector depending on the spatial coordinate $\mathbf{x}$ ($\mathbf{x} = (x, y)$ for image data) and time t. $\mathbf{X}$ is defined on $A = \Omega \times [0, \tau]$, Ω being the bounded spatial domain and $[0, \tau]$ the temporal domain.

We assume $\mathbf{X}$ is evolving in time according to:

$$\frac{\partial \mathbf{X}}{\partial t}(\mathbf{x}, t) + \mathbb{M}(\mathbf{X})(\mathbf{x}, t) = 0 \tag{1}$$

$\mathbb{M}$, named the *evolution model*, is supposed differentiable.

Observations $\mathbf{Y}(\mathbf{x}, t)$, for instance satellite image acquisitions, are available at location $\mathbf{x}$ and date t and linked to the state vector through an observation equation:

$$\mathbf{Y}(\mathbf{x}, t) = \mathbb{H}(\mathbf{X})(\mathbf{x}, t) + \mathcal{E}_O(\mathbf{x}, t) \tag{2}$$

In this paper, we assume that one component of $\mathbf{X}$ is directly comparable to $\mathbf{Y}$. Consequently, $\mathbb{H}$ reduces to a projection operator. The *observation error* $\mathcal{E}_O$ simultaneously represents the imperfection of the observation operator $\mathbb{H}$ and the measurement errors.

We consider having some knowledge on the initial condition of the state vector at $t = 0$:

$$\mathbf{X}(\mathbf{x}, 0) = \mathbf{X}_b(\mathbf{x}) + \mathcal{E}_b(\mathbf{x}) \tag{3}$$

$\mathbf{X}_b$ is named *background value* of the initial condition and $\mathcal{E}_b$ the *background error*.

$\mathcal{E}_b$ and $\mathcal{E}_O$ are assumed to be Gaussian and fully characterized by their covariance matrices $\mathbf{B}$ and $\mathbf{R}$.

2.2 Variational formulation

In order to solve the system (1), (2), (3) with respect to $\mathbf{X}$ having a maximal *a posteriori* probability given the observations, a functional $E(\mathbf{X})$ is defined and minimized:

$$\begin{aligned} E(\mathbf{X}) = &\int_A [\mathbf{Y}(\mathbf{x}, t) - \mathbb{H}(\mathbf{X})(\mathbf{x}, t)]^T \mathbf{R}^{-1}(\mathbf{x}, t)[\mathbf{Y}(\mathbf{x}, t) - \mathbb{H}(\mathbf{X})(\mathbf{x}, t)]) d\mathbf{x}dt \\ &+ \int_\Omega [\mathbf{X}(\mathbf{x}, 0) - \mathbf{X}_b(\mathbf{x})]^T \mathbf{B}^{-1}(\mathbf{x})[\mathbf{X}(\mathbf{x}, 0) - \mathbf{X}_b(\mathbf{x})] d\mathbf{x} \\ &+ Reg \end{aligned} \tag{4}$$

In this formulation, we consider no correlation of the errors between two space-time positions. *Reg* is a regularization term used to obtain a convex function and allow the minimization process to converge to a global minimum. The minimization of $E(\mathbf{X})$ is carried out with an iterative method based on the one described in [10] and summarized in the following.

At each iteration k, the analysis $\mathbf{X}_a^k$ is obtained from the background $\mathbf{X}_b^k$ by computing the increment $\delta\mathbf{X}$ at $t = 0$.

$$\mathbf{X}_a^k(\mathbf{x}, 0) = \mathbf{X}_b^k(\mathbf{x}, 0) + \delta\mathbf{X}(\mathbf{x}) \tag{5}$$

1. **Initialization**

   (a) $k = 0$

(b) Compute $\mathbf{X}_b^0(\mathbf{x}, t)$ from the initial condition $\mathbf{X}_b(\mathbf{x})$ of the state vector at $t = 0$ in (3):

$$\mathbf{X}_b^0(\mathbf{x}, 0) = \mathbf{X}_b(\mathbf{x}) \tag{6}$$

$$\frac{\partial \mathbf{X}_b^0}{\partial t}(\mathbf{x}, t) + \mathbb{M}(\mathbf{X}_b^0)(\mathbf{x}, t) = 0, \text{ for } t = 0 \text{ to } \tau \tag{7}$$

(c) Initialize the analysis $\mathbf{X}_a^0(\mathbf{x}, t)$:

$$\mathbf{X}_a^0(\mathbf{x}, t) = \mathbf{X}_b^0(\mathbf{x}, t) \ \forall t \in [0, \tau] \tag{8}$$

2. **Repeat**

(a) Compute the adjoint variable λ from $t = \tau$ to $t = 0$:

$$\lambda(\mathbf{x}, \tau) = 0 \tag{9}$$

$$-\frac{\partial \lambda}{\partial t}(t) + \left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right)^* \lambda(t) = \mathbb{H}^T \mathbf{R}^{-1}[\mathbf{Y}(t) - \mathbb{H}\mathbf{X}_a^k], \text{ for } t = \tau \text{ to } 0 \tag{10}$$

(b) Update the value of the background variable:

$$\mathbf{X}_b^{k+1} = \mathbf{X}_a^k \tag{11}$$

(c) Compute the incremental variable $\delta \mathbf{X}$ at $t = 0$:

$$\delta \mathbf{X}(\mathbf{x}) = \mathbf{B}(\mathbf{x})\lambda(\mathbf{x}, 0) \tag{12}$$

(d) Update the value of the analysis variable:

$$\mathbf{X}_a^{k+1}(\mathbf{x}, 0) = \mathbf{X}_b^{k+1}(\mathbf{x}, 0) + \delta \mathbf{X}(\mathbf{x}) \tag{13}$$

(e) Compute $\mathbf{X}_a^{k+1}(\mathbf{x}, t)$ from the initial condition:

$$\frac{\partial \mathbf{X}_a^{k+1}}{\partial t}(\mathbf{x}, t) + \mathbb{M}(\mathbf{X}_a^{k+1})(\mathbf{x}, t) = 0, \text{ for } t = 0 \text{ to } \tau \tag{14}$$

(f) $k = k + 1$

Until $||\delta \mathbf{X}||^2 \leq \varepsilon$

3. Final result is $\mathbf{X}_a^k$.

Equation (10) makes use of the adjoint model $\left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right)^*$. In our study, the discrete adjoint model is automatically obtained by the Tapenade software[1].

[1] http://www-sop.inria.fr/tropics/

3 Image models

The two *Image Models* used in the paper are based on the assumption that a pixel value is a passive tracer transported by the surface velocity field. The state vector $\mathbf{X}$ includes the motion vector $\mathbf{W}$ and a tracer q that can be directly compared to the image observations. The evolution of q is given by the advection-diffusion equation:

$$\frac{\partial q}{\partial t} + \mathbf{W} \cdot \nabla q = \nu_q \Delta q \tag{15}$$

with ν_q standing for the diffusion coefficient.

The *Stationary Image Model* (SIM) is based on the restrictive assumption that, at each position, the velocity is constant over time. The underlying hypothesis is that the surface velocity field evolves much slower than the temperature field. This heuristic is acceptable for a large range of marine processes. If a vortex, whose spatial scale is more than $10-50km$, is transported with a velocity less than 0.1 to $0.5m/s$, then the temporal scale of that phenomenon will be more than one day. It means that the surface velocity field can be considered as stationary during one day. Defining $\mathbf{X} = (u, v, q)^T$, with u and v the two components of the 2D motion vector $\mathbf{W}$, SIM is defined as: However, the stationary hypothesis makes this image model only applicable on a short temporal window.

The shallow-water equations, derived from the Navier-Stokes equations, link the 2D velocity (u, v) of the layer to its thickness h and take into account the gravity and Coriolis forces. The state vector $\mathbf{X}$ is $(u, v, h, q)^T$ and the *Shallow Water Image Model* (SWIM) is defined as: with $B = gh + \frac{1}{2}(u^2 + v^2)$, g the reduced gravity, f the Coriolis parameter (depending on the latitude), ξ the vorticity ($\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$).

4 Application of Data Assimilation

Data Assimilation is applied to perform motion estimation. The sequence of SST images $T(\mathbf{x}, t)$ is assimilated in the two models SIM or SWIM, using the incremental method described in Section 2.2.

As said in Section 2.1, the pixel value $T(\mathbf{x}, t)$ is directly comparable to the component $q(\mathbf{x}, t)$ of the state vector. The observation operator $\mathbb{H}$ reduces to a projection operator, $\mathbb{H}(\mathbf{X}(\mathbf{x}, t)) = q(\mathbf{x}, t)$. The regularization term is based on the L^2-norm of the motion gradient (to obtain a smooth vector field) and on the motion divergency (incompressibilty assumption). Its impact is analyzed in [14]. As we consider perfect models, the value of $\mathbf{X}(t)$ is obtained from the initial conditions $\mathbf{X}(0)$ by integrating in time. Hence, the cost function (4) only depends on the initial conditions and is rewritten as:

$$\begin{aligned} E(\mathbf{X}(0)) = & \int_A (T-q)^T \mathbf{R}^{-1}(\mathbf{x}, t)(T-q) d\mathbf{x} dt \\ & + \int_\Omega \big(\mathbf{X}(0) - \mathbf{X}_b\big)^T \mathbf{B}^{-1}(\mathbf{x}) \big(\mathbf{X}(0) - \mathbf{X}_b\big) d\mathbf{x} \\ & \int_\Omega \alpha(|\nabla u|^2 + |\nabla v|^2) d\mathbf{x} + \int_\Omega \beta |div\, \mathbf{v}|^2 d\mathbf{x} \end{aligned} \tag{16}$$

The choice of the covariance matrix $\mathbf{R}$ is crucial for the quality of results. As the satellite images are provided with meta-data information (see section 5.1), the quality of the acquisitions is approximately known. $\mathbf{R}^{-1}(\mathbf{x}, t)$ is then given a small value when the acquisition is noisy at $(\mathbf{x}, t)$ (because of cloud occlusion for instance). The choice of the initial background conditions has also a strong impact on the quality of the result. It has been discussed in [14] that the best results are obtained with the first observation as background for q, null value for $\mathbf{W}$ and a constant value h_m for h, with h_m being the thickness value at rest state. As the background value of q is reliable, $\mathbf{B}_q$ is given a small value.

5 Results

5.1 Image data

A huge amount of images are acquired over the ocean by space remote sensors. Those obtained by optical instruments, such as *Sea Surface Temperature* (SST) data, display a high space-time coherence. The images, used in the paper, are acquired on-board NOAA-AVHRR satellites. Their spatial resolution is 1.1 km^2 at nadir and the temporal revisit is at best one day. However, several acquisitions over the same area are usually acquired on the same day by different satellites. Some of these data are contaminated by clouds or corrupted by noise. Figure 1 displays a SST image acquired over the Black Sea in October 2005, with the cyan color corresponding to clouds or noise.

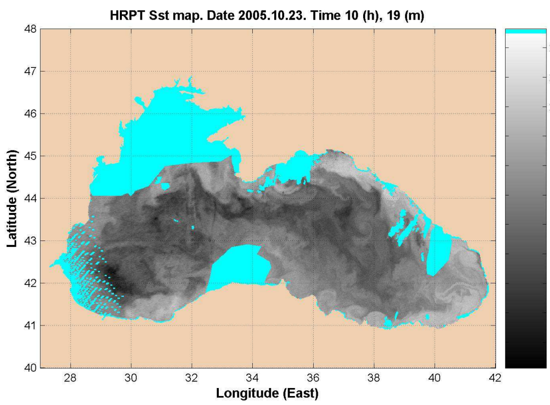

Figure 1: Cyan area corresponds to clouds or noise.

5.2 Analysis

In this paper, motion estimation is tested on a sequence of four images, displayed on Figure 2. The cyan areas, on the third and fourth frames, correspond to missing data.

The two *Image Models* are used to estimate the surface velocity on these data. Figure 3 compares the motion fields estimated with SIM and SWIM, at $t = 0$. The results obtained with SWIM visualize a cyclonic vortex on the western

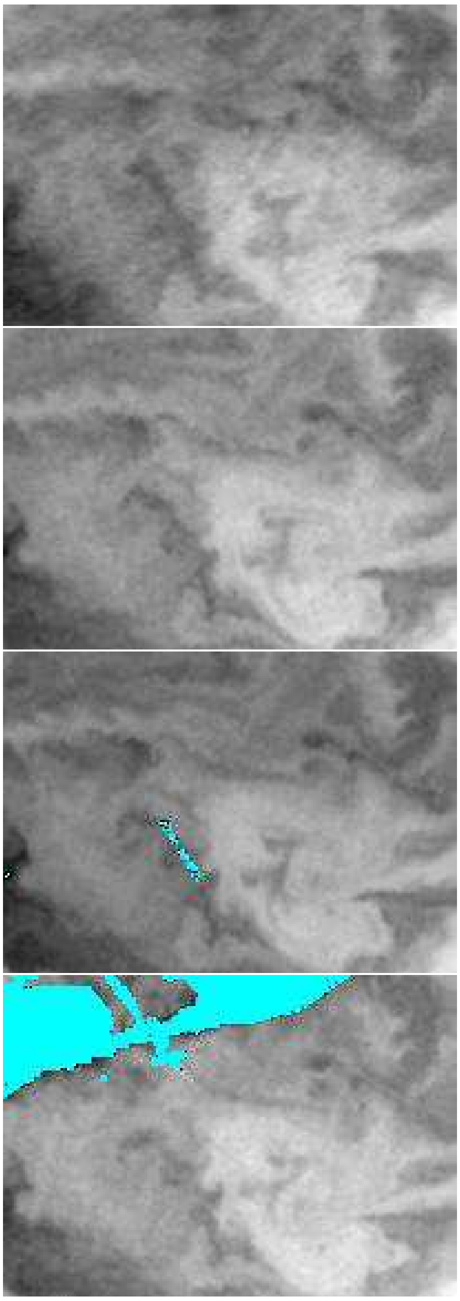

Figure 2: SST data acquired from October 23th to October 24th, 2005.

part of the Black Sea. SWIM, due to its physical assumptions on the dynamic, permits a more realistic motion estimation and characterizes structures occuring on the sea surface. In comparison, the potential of SIM highly depends on the size of the temporal window compared to the dynamics involved during that period. That makes SIM no more relevant for data such as those displayed in Figure 2. In conclusion, the DA approach for motion estimation permits to

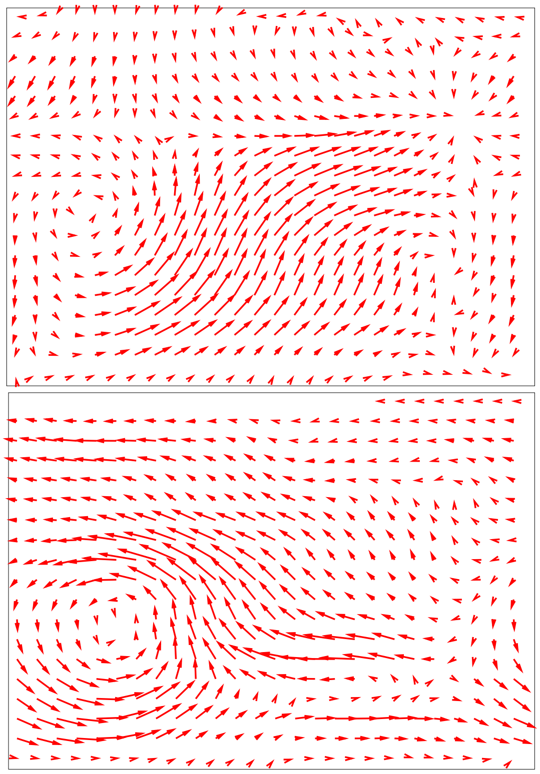

Figure 3: Motion estimation. Up: SIM; down: SWIM.

retrieve the major currents of the Black Sea basin. Moreover, the high resolution of NOAA/AVHRR images allows to better evaluate the size of some well known mesoscale structures [12].

5.3 Altimetry data

Satellite altimeters provide an accurate measure of the Sea Level Anomaly (SLA) that corresponds to the sea surface deviation from its rest state (see the black curve on Figure 4).

The altimeters are nadir-pointing instruments providing an along-track acquisition. The coverage of Envisat1 over the Black Sea is for instance displayed on Figure 5. In this paper, we use altimetry measures provided by Envisat[2] with a 35 days cycle and by GFO [3] with a 17 days cycle.

[2]http://envisat.esa.int

[3]http://ilrs.gsfc.nasa.gov/satellite_missions/list_of_satellites/gfo1_general.html

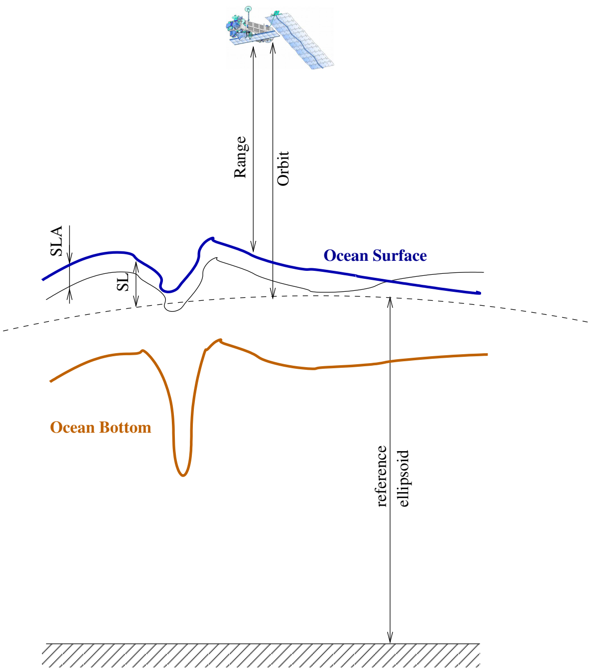

Figure 4: Sea Level Anomaly.

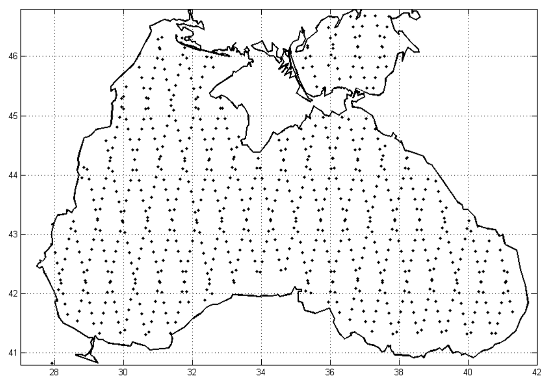

Figure 5: 35 days cycle of Envisat1.

5.4 Validation

The outputs of SWIM are $\mathbf{W}$, the surface velocity, and h the thickness of the surface layer. The thickness anomaly, denoted h_{SWIM}, is estimated from h as its deviation from the value at rest. On another hand, the altimeters are 1-dimensional instruments measuring the Sea Level Anomaly, denoted h_{alt}, along their tracks. We then compare h_{SWIM} and h_{alt} at the same positions. The physical formula linking these two quantities is:

$$\rho \times h_{alt} = \Delta\rho \times h_{SWIM} \tag{17}$$

with ρ being the density of the upper layer, $\Delta\rho$ the difference of density between the upper and the lower layer, h_{alt} the sea level anomaly measured by the satellite, h_{SWIM} the thickness anomaly $(h - h_m)$ of the shallow-water model.

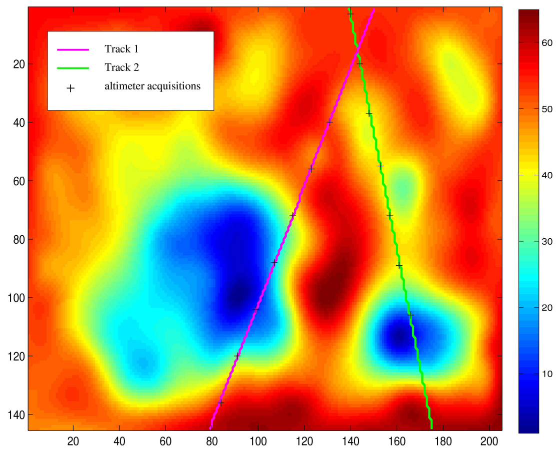

Figure 6: Two altimeter tracks displayed over the average of h_{SWIM}.

Figure 6 displays the value of h_{SWIM}, averaged in time. The two straight lines represent altimeter tracks. The green line comes from Envisat and the pink one from GFO.

The number of altimetry measures available on the same space-time period than the SST data is rather small. However, we apply the conversion given in (17) and perform a quantitative comparison of h_{alt} and h_{SWIM} along a track. Figure 7 displays these curves for the two tracks displayed on Figure 6: on the left with Envisat and on the right with GFO. Black crosses locate the altimeter measures. The shapes and values of h_{alt} and h_{SWIM} curves are very similar. There is no error in the slope directions. The extrema are well localized. It is almost perfect in the case of Envisat. As the velocity field is strongly related to the shape of the thickness image, these promising results on thickness estimation validate the estimation of the motion. Figure 8 illustrates the link between velocity and thickness: a bump correspond to an anticyclonic velocity field and a bowl to a cyclonic one.

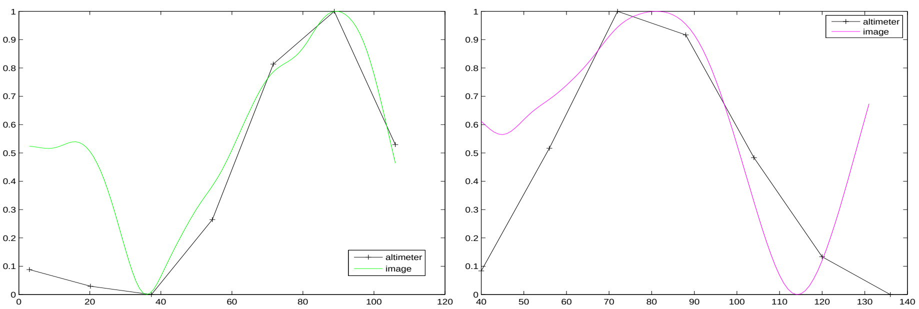

Figure 7: Sea Level Anomaly, given by the altimeters, compared with the estimation with SWIM.

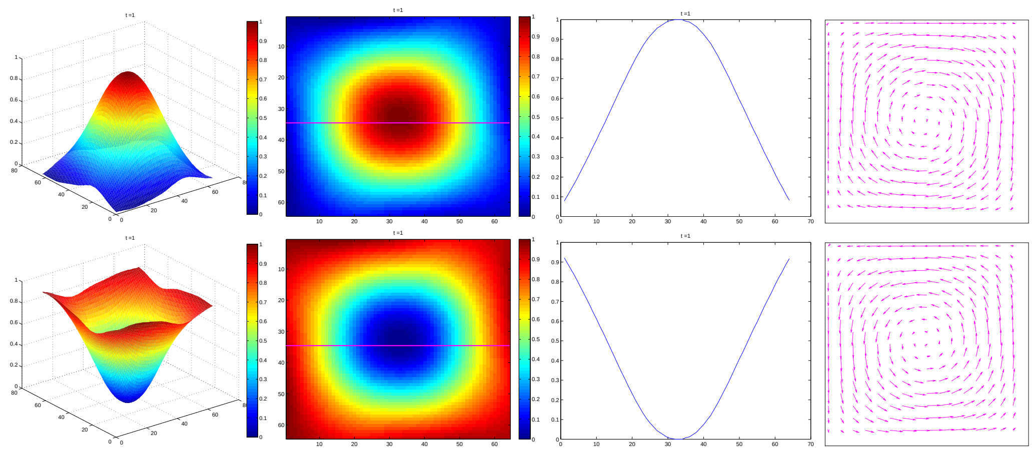

Figure 8: From left to right. 1. 3D water layer thickness. 2. Its 2D projection. The magenta line figures the track of an altimeter. 3. SLA along this track. 4. Velocity field.

6 Conclusion

In this paper, we propose an analysis and validation of the data assimilation approach for motion estimation from satellite image sequences. We compared two dynamic assumptions, *i.e.* we assimilated the same data in two image models, SIM and SWIM, and analyzed motion results. Moreover, we used altimetry data to quantify the quality of the estimation. The comparison between the surface anomaly estimated by SWIM and measured by altimeters validates our approach.

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